Inspiring Greatness

INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

AP Stats / Pairs T-test

April 27, 2020

Lesson: April 27, 2020

Objective/Learning Target:
Students will be able to identify when a pairs t-test vs a 2-sample t-test is an appropriate method.

Review #1

NASA wishes to know the effects of space travel on the human body. They manage to somehow find 5 pairs of twins that are all astronauts! They randomly assign one of each twin to the crew. The others to a control group. All five astronauts and their twins undergo extensive medical tests. Then the astronauts spend a year in space working on the International Space Station. Upon their return, both the astronauts and their twins undergo another round of medical tests. NASA then compares the change in health of both the astronauts and their twins to determine what changes are likely to be caused by space travel.

What kind of study is this? Can we consider the group of astronauts and twins two independent samples?

Review #2

Wanting more evidence, NASA attempts a new study to determine the effect of space travel on astronauts. They gather 100 potential astronauts. Half of them are randomly assigned to spend a year in space and the other half are randomly assigned to stay on Earth. All of them are assigned a specific diet and exercise routine that can be done equally well on Earth and space. The change in health is measured for each group and compared.

- What type of study is being described? Are the two groups independent of each other?
- What change could be done to possible reduce the variation between groups?

Answers

1. This is a matched pairs experiment. Each astronaut is paired with their twin. When using twins these are also called twin studies. NASA actually did this with a single twin… imagine the difficulty of finding 5 twins that can all become astronauts! The two groups **cannot be considered independent**! The genetics within each pair are practically identical and thus each pair has a higher probability of acting similarly. If one twin is susceptible to an illness, the twin is also more likely to be susceptible too. Thus our 2 sample and 2 prop t-tests are no longer valid.

Answers

2. This would be a completely randomized design. There is no sorting or pairing of individuals. Since individuals in each group are randomly assigned, we can assume that the individuals in each group are independent of each other. One person getting an illness does not affect the others. We would want to avoid having close relatives involved in the study to preserve independence.

We could change this study to a blocked design. It seems reasonable that different genders could react differently, so blocking by gender could reduce variability. Other variables known to change the way bodies are affected (family health history, chronic conditions, weight, etc.) could be good options for blocking as well.

How do we deal with violations of independence?

The first review question has two groups that are not independent. The health of each astronaut is related to their twin. One getting an illness means their twin is more likely to get the same illness.

If we wanted to estimate the average decrease in bone density over a year in space, we could not run a 2 sample t-test on our results of the twin study. Remember that the 2 sample t-test assumes that we are using independent samples. This violation of our assumptions means the test is not valid… so how can we analyze these results?

We instead find the difference between each pair and run a 1 sample t-test on the differences.

Example

Below are bone density measures for the 5 pairs of twins. Do we have evidence of decreased bone density from space travel?

Astronaut	-0.3	-0.5	-0.3	0.9	-2.1
Control	0	0.2	0.2	0.4	-1.1

State

We want to test to see the difference in bone density loss between the astronauts and their twins is greater in space.

μ: The mean of the earth bound twin's bone density minus the astronaut twins bone density for each pair

$$H_0: \mu = 0$$

$$H_a: \mu > 0$$

Here we define one mean by first taking the difference in each pair, then finding the mean of those differences. This contrasts with the 2 sample t-test that finds each mean individually then finds the differences of the means.

Plan

We will use a paired samples t-test with $\alpha = 0.05$

Assumptions:

Random: One of each pair is randomly assigned to be an astronaut, the other stays on earth as a control.

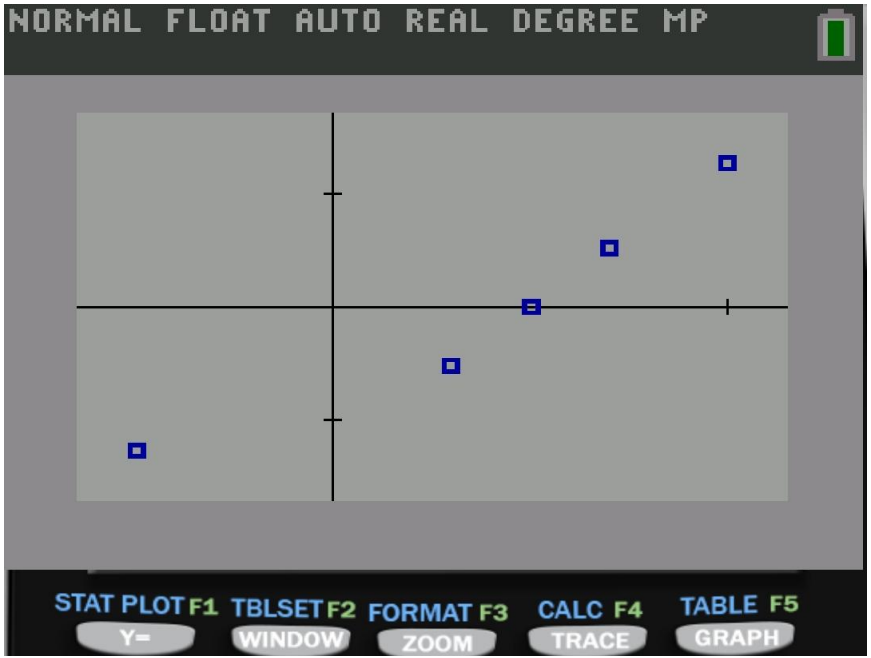
Independent: The results of each pair of twins should not affect the results of other twins. Additionally, it seems reasonable the 5 pairs of twins is less than 10% of all twins.

Normal: we need some graphs!

Normal

We can examine boxplots, histograms, and several other plots. Here we are examining a normal probability plot.

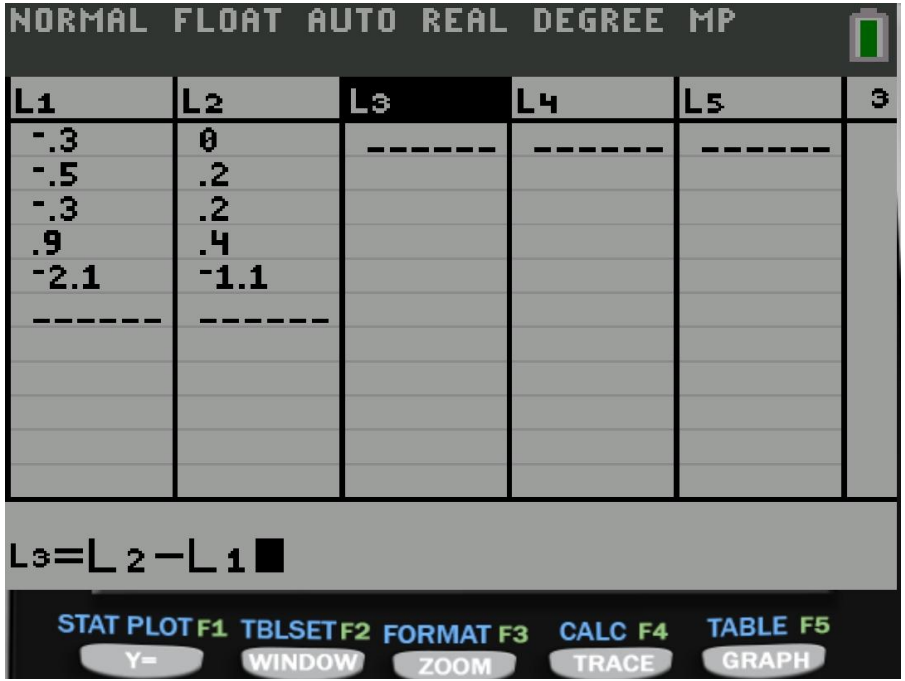
The plot shows an approximately linear pattern, with only one point slightly of linear but it is near the end of the plot. With the small sample size, we will assume this is approximately normal. Thus by the central limit theorem the sampling distribution should also be normal.



Do - Using a calculator!

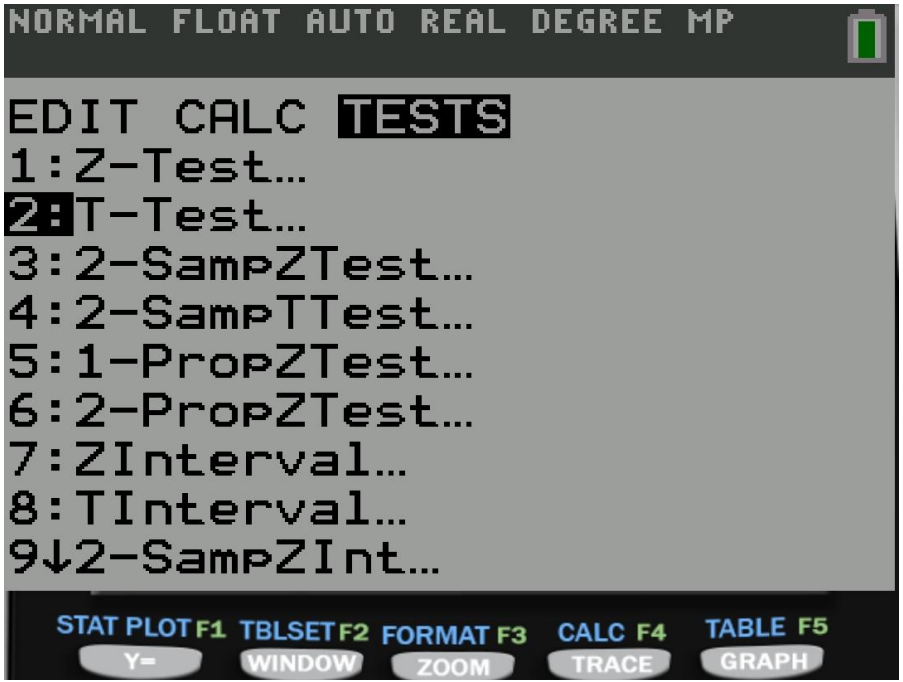
This step is simply the 1 sample t-test on the differences. We are going to do it on the calculator.

We will use the lists to do some math for us. Enter the astronauts in list 1 and control in list 2. Then scroll up to highlight L_3. Then type L_2-L_1 and press enter. You now have a list of the differences!



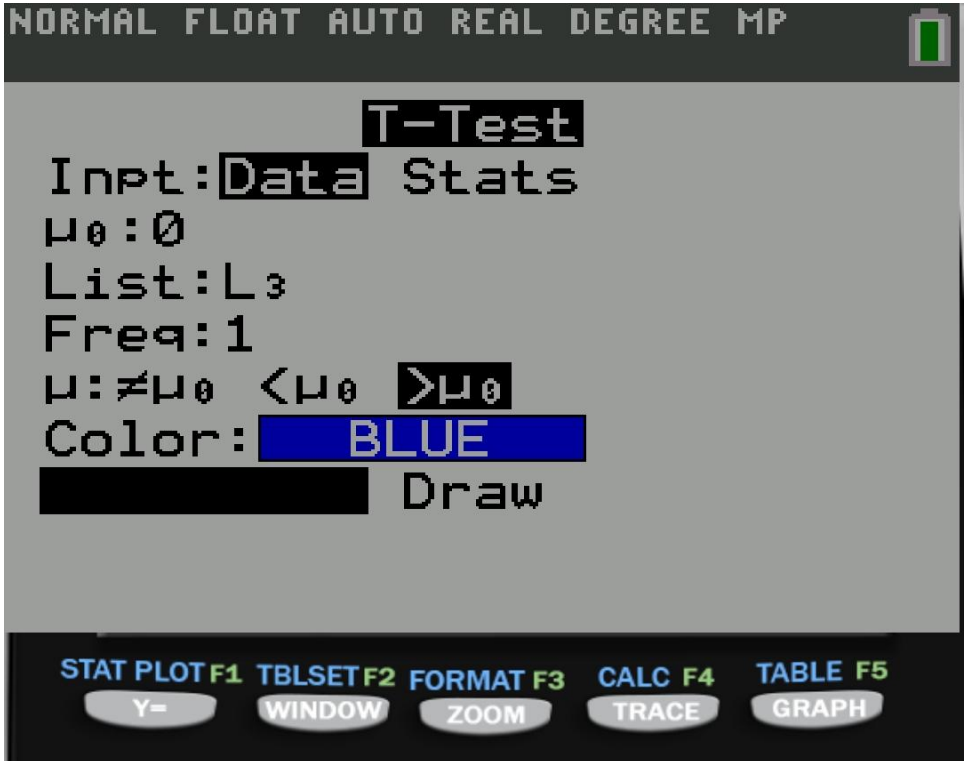
Do - Using a calculator!

Now that L_3 has a list of the differences. Press stat, scroll to tests and choose T-Test.



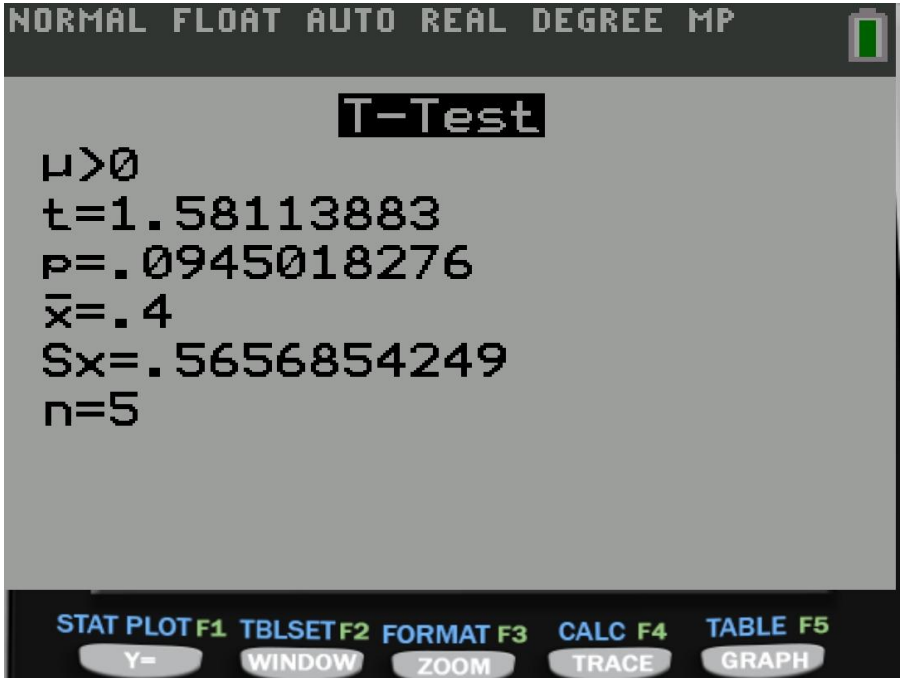
Do - Using a calculator!

We are going to use the data straight from the list. Set the null mean to 0, list to L_3 and the alternative to greater than. Press calculate.



Do - Using a calculator!

This gives us the results of the test. Make sure to state the calculator and function that you used. Then copy down all outputs.



Conclude

There is not sufficient evidence to reject the null hypothesis at $\alpha = 0.05$, with a p-value of 0.0945. We therefore cannot conclude that space travel results in a decrease of bone density.

Critical thinking: If we assume the true difference between the two is 0.2. That is space travel causes an average decrease in bone density of 0.2. Why would the test not be able to detect this change? What type of error is this?

Extra Practice

Free Response Problem

Answers